IONIZATION-OVERHEATING INSTABILITY IN A LOW-VOLTAGE ARC DISCHARGE

V. A. Zherebtsov and I. P. Stakhanov

We consider the stability of a layer of weakly ionized plasma between conducting electrodes closed by an external circuit. It is assumed that the plasma ions are formed due to space ionization, and the electrons are formed as a result of emission from the heated electrode (cathode). Such a system is called a low-voltage arc or an arc with heated cathode. The volt–ampere characteristics of a low-voltage arc have a region of negative resistance in which various instabilities of the overheating type can develop. Overheating instabilities have been investigated in a number of papers for semiconducting and gas-discharge plasmas [1-3]. As distinct from the cases previously considered, the mechanism of overheating instability is closely associated with the processes of space ionization which occur in the discharge. In this paper we obtain a nonlinear equation describing the nonstationary low-voltage arc discharge which is inhomogeneous along the electrodes. On the basis of this equation we investigate the stability of a homogeneous discharge. We show that when the differential resistance is negative the discharge is unstable, and when it is positive the discharge is stable. The development of perturbations which are homogeneous along the electrodes leads to an overthrow of the discharge into that part of the characteristics in which they are stable. Inhomogeneous perturbations lead to the formation of a transverse inhomogeneous structure in the discharge or a local reduction of the current density.

1. Let the origin of coordinates be chosen at the middle of the plasma layer under consideration; the x axis directed perpendicular to the electrodes and the y and z axes parallel to them. The distance $2l$ between the electrodes is much less than their transverse dimensions, which are assumed to be unbounded. As we know, in a low-voltage arc the plasma is separated from the electrodes by potential barriers which restrict the electron flux from it (Fig. 1). As a result of this, and also because of the high thermal conductivity of the electrons, we can assume that the temperature of the electron gas, T_e, is independent of x.

We shall also assume that T_e is not a function of the time or the transverse coordinates in all terms with the exception of the ionization rate coefficient α, which is very sensitive to temperature changes ($\alpha \sim \exp(-E_i/T_e)$, $E_i/T_e \sim 20$). The ion temperature T_i, which coincides with the temperature of the atoms, is also assumed to be constant. We consider systems of such dimensions that we can ignore energy exchange between electrons and ions, i.e., $T_e \neq T_i$.

The electron J_x and ion j_x flux densities can be expressed by the following equations:

$$J_x = -D_e \frac{dn}{dx} + nu_e \frac{d\varphi}{dx} \tag{1.1}$$

and a similar equation for the ions, where the diffusion coefficients D_e and D_i and the mobility coefficients u_e and u_i are determined by collisions with neutral atoms and are assumed to be constant. In view of the intense volume ionization in a low-voltage arc,

$$J_x \gg j_x \gg J_x u_i / u_e \tag{1.2}$$

and so

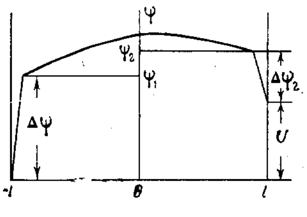

Fig. 1

Obninsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 35-44, May-June, 1971. Original article submitted June 8, 1970.

$$j_x = -\frac{u_i}{u_e} J_x - D_a \frac{dn}{dx} \approx -D_a \frac{dn}{dx}, \qquad D_a = \left(1 + \frac{T_e}{T_i}\right) D_i \tag{1.3}$$

Here D_a is the ambipolar diffusion coefficient. It follows from the equation of continuity for ions that

$$\frac{\partial n}{\partial t} - D_a \frac{\partial^2 n}{\partial x^2} = \alpha n \tag{1.4}$$

Here αn is the number of ions originating in unit volume in unit time.

The stationary density distribution has a maximum [4] near the plane x = 0. Hence, in what follows, to a sufficiently high degree of accuracy, we can assume that the solution of (1.4) is symmetrical and satisfies the following boundary conditions:

$$D_a \frac{dn}{dx}\bigg|_{x=\pm l} = \mp \frac{1}{2} v_i n_1 \qquad \left(v_i = \left(\frac{8T_i}{\pi M}\right)^{1/2}\right) \tag{1.5}$$

Here n_1 is the plasma density at the cathode ($x = -l$) or the anode ($x = l$). The coefficient $1/2$ on the right side of (1.5) is chosen so as to take account of the departure from equilibrium of the ion distribution due to the effect of the electrode.

As a result, in the stationary case,

$$n = n_1 \cos \gamma x / \cos \gamma l \qquad (\gamma = \sqrt{\alpha / D_a}) \tag{1.6}$$

It was shown in [4] that

$$\delta \gamma l \operatorname{tg} \gamma l = 1, \quad \gamma l = {}^1/_2 \pi (1 - \delta), \quad \delta = 2D_a / v_i l \ll 1 \tag{1.7}$$

It follows from the foregoing results that the ion flux is symmetrical to both electrodes, while the electron current J_x flowing from the cathode to the anode is small by comparison with each of the components separately on the right side of (1.1).

2. The problem below is to obtain a closed system of equations describing the nonstationary processes in a low-voltage arc. We shall assume that the discharge can be inhomogeneous with respect to y and z. In accordance with the considerations which follow, we can considerably simplify the problem under discussion.

We note that the set of solutions of the equation

$$\frac{d^2 n_k}{dx^2} = -\gamma_k^2 n_k \tag{2.1}$$

with the boundary conditions (1.5) forms a complete orthonormalized system of functions in the interval $-l < x < l$, the eigenvalues of which are determined from the solution of the transcendental equations

$$\delta \gamma_k l \operatorname{tg} \gamma_k l = 1 \quad \text{for } k = 1, 3, 5, \ldots; \qquad -\delta \gamma_k l \operatorname{ctg} \gamma_k l = 1 \quad \text{for } k = 2, 4, 6, \ldots \tag{2.2}$$

The completeness and orthogonality of this system follow from the self-conjugacy of the operator (2.1). As $\delta \to 0$ the solutions of (2.2) transform to the form

$$\gamma_k l = {}^1/_2 \pi k \quad (k = 1, 2, 3, \ldots) \tag{2.3}$$

We seek the solution of the nonstationary plasma diffusion equation

$$-\frac{\partial n}{\partial t} + D_a \frac{\partial^2 n}{\partial x^2} + D_a \Delta_\perp n + \alpha n = 0, \qquad \Delta_\perp = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{2.4}$$

in the form of a series

$$n = \sum_{k=1}^{\infty} N_k (t, y, z)\, n_k (x) \tag{2.5}$$

Then, for the coefficients of that series we have

$$-\partial N_k / \partial t + D_a \Delta_\perp N_k + (\alpha - \alpha_k) N_k = 0 \qquad (\alpha_k = \gamma_k^2 D_a) \tag{2.6}$$

It is easy to see that in the stationary and homogeneous (with respect to y,z) state the unique positively defined solution of Eq. (2.6) in the interval $-l < x < l$ is

$$\alpha = \alpha_1, \quad N_1 \neq 0, \quad N_k = 0 \text{ for } k \neq 1 \tag{2.7}$$

This corresponds to the results (1.6) and (1.7). Further, from (2.6) we see that if, in the nonstationary state, the value of α is less than α_2, then, for all k, apart from k = 1, N_k is exponentially damped, and thus the general solution of (2.5) reduces to the solution with separated variables

$$n = N_1(t, y, z) \cos \gamma x = n_1(t, y, z) \cos \gamma x / \cos \gamma l \tag{2.8}$$

where $\gamma = \gamma_1$ is defined by (1.7). The next term in the series occurs only when α is made greater than $\alpha_2 \approx 4\alpha_1$. Thus, there is a wide region of nonstationary states in which the solution can be represented as (2.8). In what follows we restrict ourselves to just this region.

If we integrate Eq. (2.4) with respect to x from $-l$ to l, and note that, as will be shown below, the term $D_a \Delta_\perp n$ does not make a significant contribution to the final result, we find that

$$\partial N / \partial t + v_i n_1 - \alpha N = 0, \qquad N = \int_{-l}^{l} n dx \tag{2.9}$$

It follows from (2.8) that

$$n_1 = bN, \qquad b = \frac{\pi^2}{4} \frac{D_a}{v_i l^2} \tag{2.10}$$

Thus,

$$\frac{\partial N}{\partial t} + (\beta - \alpha) N = 0, \qquad \beta = \frac{\pi^2}{4} \frac{D_a}{l^2} \tag{2.11}$$

To obtain a closed system of equations we compute the potential drop in the interelectrode space. It follows from (1.1) that

$$\psi = \psi_1 + \ln \frac{n}{n_1} + \frac{1}{D_e} \int_{-l}^{x} \frac{J_x}{n} dx \qquad \left(\psi = \frac{e\varphi}{T_e}\right) \tag{2.12}$$

The pre-electrode potential discontinuity ψ_1 at the cathode and $\Delta\psi_2$ at the anode are determined from the equations

$$J_1 = J_R - {}^1/_4 n_1 v_e \exp(-\psi_1), \quad J_2 = {}^1/_4 n_1 v_e \exp(\Delta\psi_2) \tag{2.13}$$

Here J_R is the flux density of electron emission and $J_{1,2}$ are the flux densities of the electrons at the cathode and the anode. It follows from (2.12) and (2.13) that the total potential drop in the discharge, U, is (in units of T_e/e)

$$U \equiv \psi_1 + (\psi_2 - \psi_1) + \Delta\psi_2 = \ln \frac{J_2}{J_R - J_1} + \frac{1}{D_e} \int_{-l}^{l} \frac{J_x}{n} dx \tag{2.14}$$

In what follows we shall assume that the derivatives along the electrodes are small by comparison with the derivatives with respect to x. This implies that

$$\delta J = J_2 - J_1 \ll J_2 \tag{2.15}$$

Further, in the integral on the right side of (2.14), the basic contribution comes from the regions near the electrodes, where the plasma density is small. As a result,

$$\int_{-l}^{l} \frac{J_x}{n}\,dx = \frac{J_1+J_2}{2}\int_{-l}^{l}\frac{dx}{n} \tag{2.16}$$

For the density distribution (2.8), the integral (2.16) is

$$\int_{-l}^{l}\frac{dx}{n} = \frac{d}{N}, \qquad d = \frac{16}{\pi^2} l^2 \ln\left(\frac{4}{\pi\delta}\right) \tag{2.17}$$

If we expand the first term on the right side in (2.14) in terms of δJ and use (2.16), (2.17), we obtain

$$U - \ln\frac{I}{1-I} = \frac{I}{\nu} - \left(\frac{1}{1-I} + \frac{1}{2\nu}\right)\delta I, \quad I = \frac{J_2}{J_R}, \quad \delta I = \frac{\delta J}{J_R}, \quad \nu = \frac{D_e N}{dJ_R} \tag{2.18}$$

For small δI the term $\ln[I/(1-I)]$ is the total voltage drop in the pre-electrode layers, I/ν is the potential drop in the plasma, the term $\sim\delta J$ is the change in the potential drop due to currents flowing along the electrodes.

We consider now the energy balance in the electron gas

$$\operatorname{div}\mathbf{q} = -\alpha n E_i, \qquad \mathbf{q} = \mathbf{J}(2T_e - e\varphi) - 2D_e n\nabla T_e \tag{2.19}$$

Here E_i is the effective energy lost by the electron gas in one ionization act. The energy fluxes entering the electron gas at the cathode and anode, respectively, are [4]

$$q_1 = 2T_e J_1 - 2(T_e - T_1)J_R, \quad q_2 = (2T_e - eV)\,J_2 \qquad \left(V = \frac{T_e}{e}U\right) \tag{2.20}$$

Here T_1 is the cathode temperature. In evaluating (2.20), we assume that the cathode surface is at zero potential. Integrating (2.19) with respect to x from $-l$ to l and using (2.20) and the equation of continuity for electrons, $\operatorname{div}\mathbf{J} = 0$, we obtain

$$UI = a + \varepsilon\varkappa\nu - \nabla_\perp \int_{-l}^{l}\frac{\mathbf{J}_\perp}{J_R}\psi\,dx \tag{2.21}$$

$$a = 2\left(1 - \frac{T_1}{T_e}\right), \quad \varepsilon = 4\frac{D_a}{D_e}\frac{E_i}{T_e}\ln\left(\frac{4}{\pi\delta}\right) \ll 1, \quad \varkappa = \frac{\alpha}{\beta}$$

where $\mathbf{J}_\perp$ is the projection of the vector $\mathbf{J}$ on the yz plane. We see from (2.21) that the energy introduced by the current into the interelectrode space (IU) is expended on heating the electron beam leaving the cathode to temperature T_e (coefficient a), on ionization ($\varepsilon\varkappa\nu$), and on the creation of an energy flux along the electrodes carrying the transverse current. Because the energy of the electrons is small, in computing the energy balance and the voltage drop we can neglect the time derivatives.

3. Consider a discharge which is homogeneous with respect to y and z. In this case $\nabla_\perp = 0$, $\delta I = 0$, and Eqs. (2.11), (2.18), and (2.21) can be written as

$$\frac{d\nu}{d\tau} + (1-\varkappa)\nu = 0, \quad IU - a = \varepsilon\varkappa\nu, \quad U - \ln\frac{I}{1-I} = \frac{I}{\nu} \tag{3.1}$$

where $\tau = \beta t$. In the stationary case $\varkappa = 1$, and

$$F(U, I) \equiv \left(U - \frac{a}{I}\right)\left(U - \ln\frac{I}{1-I}\right) = \varepsilon \tag{3.2}$$

Equation (3.2) is the volt–ampere characteristic of the discharge since, by (1.7), the electron temperature T_e is independent of the current [4]. For $\varepsilon \ll 1$, it can be approximated by the following two equations:

$$f \equiv U - \frac{a}{I} = 0, \qquad g \equiv U - \ln\frac{I}{1-I} = 0 \tag{3.3}$$

The first of these gives a branch with negative differential resistance.

In the nonstationary case, Eq. (3.1) forms a closed system for the variables ν, $\varkappa$, I. The voltage drop U is associated with I through the equation

$$U = E - rI \quad r = e^2 RSJ_R / T_e \tag{3.4}$$

where E is the emf in the external circuit, r is the nondimensional resistance of the external circuit, eSJ_R is the emission current from the cathode. If we eliminate $\varkappa$ and ν from (3.1) and use (3.4), we obtain

$$\frac{dI}{d\tau} = \frac{I}{\varepsilon} \frac{F(U, I) - \varepsilon}{Gg + rI} g \qquad \left(G = 1 + \frac{1}{g(1 - I)}\right) \tag{3.5}$$

The functions F and g were defined above.

For small deviations from the equilibrium state

$$I = I_0 + I', \quad U = U_0 - rI' \tag{3.6}$$

the function F(U, j) can be written as

$$F(U, I) = F(U_0, I_0) + \left(\frac{\partial F}{\partial I}\right)_U I' - \left(\frac{\partial F}{\partial U}\right)_I rI' \tag{3.7}$$

The derivative $(\partial F / \partial I)_U$ can be expressed in terms of the differential resistance $r_d = (\partial U / \partial I)_{F=\varepsilon}$. Differentiating the equation F = const, we obtain

$$\left(\frac{\partial F}{\partial I}\right)_U = -\left(\frac{\partial F}{\partial U}\right)_I \left(\frac{\partial U}{\partial I}\right)_F = -(g + f) r_d \tag{3.8}$$

Noting that $F(U_0, I_0) = \varepsilon$ and linearizing (3.5) with the aid of (3.7) and (3.8) we can obtain

$$\frac{dI'}{d\tau} = -(r + r_d) \frac{gI_0}{\varepsilon} \frac{f + g}{Gg + rI_0} I' \tag{3.9}$$

We see from Eq. (3.9) that, since f, g, and G are positive, when $-r_d < r$, the discharge is stable, while when $-r_d > r$, instability develops. A similar result was obtained in [5], where certain simplifying assumptions in this paper were not made (in particular, assumptions about the symmetry of the plasma density distribution in the layer).

We turn to an analysis of the nonlinear equation (3.5). The volt–ampere characteristics of the discharge (3.2) are given in Fig. 2 for $\varepsilon = 0.1$, $a = 1$ (curve 1) and loading characteristic (3.4). Curves 3 and 4 represent the functions $f = 0$ and $g = 0$. Below curve 3 and above curve 4 there are forbidden regions, since it follows from (3.1) that $\nu < 0$ there. Below the curve 3 the energy introduced into the discharge (UI) is not sufficient to heat the electrons leaving the cathode to the temperature T_e. Hence the discharge is suppressed on this curve. The region above curve 4 can only be reached if there are emf sources in the plasma.

As we can see from (3.5) (cf. also Fig. 2), on the right of the volt–ampere characteristic $dI/d\tau > 0$, while on the left $dI/d\tau < 0$. The load line (3.4) intersects the volt–ampere characteristics at two points, the upper point of intersection B being always stable, while the lower A is unstable. Any current perturbations at A lead (depending on the sign of the perturbation) either to quenching of the discharge or to transition to the point B of stable equilibrium. Since this conclusion is obtained from a nonlinear equation, it holds not only for infinitely small, but also for finite perturbations.

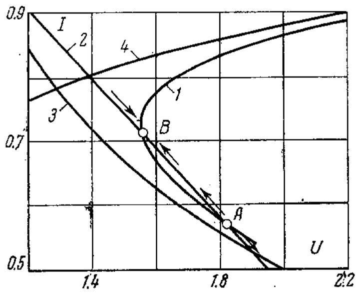

Fig. 2

To explain the instability we can assume the following physical mechanism. For simplicity we put r = 0. As a result, let the random fluctuation in the density ν increase slightly. This leads to a reduction in the potential drop in the plasma volume I/ν by an amount $-I\Delta\nu/\nu^2$. Since U must remain constant, by (3.1) the current through the discharge must increase by an amount

$$\Delta I = \frac{I^2(1 - I)}{I(1 - I) + \nu} \frac{\Delta \nu}{\nu}$$

In turn, this leads to an increase in the energy introduced into the plasma by an amount $U\Delta I$. If this were greater than the increase in the ionization losses $\varepsilon\Delta\,\nu$ [cf. Eq. (3.1)], the density fluctuations would increase and the system would depart from its initial state. Thus, for instability it is necessary that

$$\frac{UI}{\nu}\frac{I(1-I)}{I(1-I)+\nu} > \varepsilon$$

As a result of this, in the lower part of the characteristic [for $\nu \ll I(1-I)$], where the ionization losses are small and the potential drop in the plasma is large, instability occurs. Quite a different situation is created in the upper part of the characteristic [$\nu \gg I(1-I)$]. There the ionization losses are large, while the role of the potential drop in the plasma is insignificant, and is easily compensated by a small change in the current. Thus, the upper part of the characteristic is stable. A similar process also occurs when there is a fluctuating reduction in the density. The criterion obtained above coincides with the condition for a positive increment in Eq. (3.9).

4. Consider the behavior of perturbations which are inhomogeneous with respect to y and z, when the characteristic dimension of the inhomogeneity is much greater than the distance between the electrodes ($2l$). In this case, the terms in Eqs. (2.18) and (2.21) containing δI and $\nabla_\perp$ are small; a similar statement can also be made about the equation

$$\mathbf{J}_\perp = -D_e\nabla_\perp n + u_e n\nabla_\perp\varphi = -D_e[\nabla_\perp n - n\nabla_\perp\psi] \tag{4.1}$$

Further, it follows from (2.13) that

$$\psi_1 = \ln\frac{n_1 v_e}{4(J_R - J_1)} \tag{4.2}$$

In substituting (2.12) and (4.2) into (4.1) we can assume that $J \approx J_1 \approx J_2$. Then, noting that the electrodes are equipotential ($\nabla_\perp U = 0$), we can obtain

$$\mathbf{J}_\perp = D_e n\left(1 - \frac{\nu}{I}\frac{J_R}{D_e}\int_{-l}^{x}\frac{dx}{n}\right)\frac{\nabla_\perp I}{1-I} \tag{4.3}$$

In deriving (4.3) we used (2.8) and (2.18), in which, in this case, we can put $\delta I = 0$. Noting that $\operatorname{div}\mathbf{J} = 0$, from (4.3) we can obtain

$$\delta J = -D_e\nabla_\perp\left[\frac{\nabla_\perp I}{1-I}\int_{-l}^{l} n\left(1 - \frac{\nu}{I}\frac{J_R}{D_e}\int_{-l}^{x}\frac{dx}{n}\right)dx\right]$$

Since the density n has a maximum at x = 0, the term in parentheses can be taken outside the integral sign for x = 0. Then

$$\delta J = -D_e\nabla_\perp[Np(I)\nabla_\perp I], \qquad p(I) = \frac{2I-1}{2I(1-I)} \tag{4.4}$$

The same result is obtained if the integral is evaluated exactly. Similarly, we can evaluate the integral

$$\int_{-l}^{l}\mathbf{J}_\perp\psi\,dx = D_e\left[\psi_1 N + \int_{-l}^{l} n\ln\frac{n}{n_1}dx\right]p(I)\nabla_\perp I \tag{4.5}$$

The last term on the right side of (2.12) is less than the two other terms. Hence, when we substitute (2.12) in the integral (4.5) this term can be omitted. When we evaluated (4.4) the term was retained since its derivative makes an appreciable contribution [the factor $(2I-1)/2I$ instead of 1]. We can evaluate the integral on the right side of (4.5):

$$\int_{-l}^{l} n\ln\frac{n}{n_1}dx = \frac{1}{\gamma^2}\int_{-l}^{l}\frac{1}{n}\left(\frac{dn}{dx}\right)^2 dx = N\left[\ln\left(\frac{4}{\pi\delta}\right) - 1\right] \tag{4.6}$$

Thus, from (2.11), (2.18), (2.21), (4.4), (4.5), and (4.6) we obtain the following system of equations:

$$\frac{\partial\nu}{\partial\tau} + (1-\varkappa)\nu = 0 \tag{4.7}$$

$$U - \ln\frac{I}{1-I} = \frac{I}{\nu} + \frac{1}{1-I}\nabla_\perp'[p(I)\nu\nabla_\perp' I] \qquad (\nabla_\perp' = \sqrt{d}\nabla_\perp) \tag{4.8}$$

$$UI - a = \varepsilon\varkappa\nu - \nabla_\perp'[p(I)(\psi_1 + c)\nu\nabla_\perp' I] \qquad (c = \ln(4/\pi\delta) - 1) \tag{4.9}$$

The variables ν, $\varkappa$, τ, ε are defined above. In deriving (4.8) the second term in parentheses in (2.18) was omitted by comparison with the first, since $\nu > 1$ over a large part of the volt-ampere characteristic. If we take account of transverse diffusion in (2.9), the following term appears in (4.7):

$$[4\ln(4/\pi\delta)]^{-1}\Delta_\perp'\nu$$

which is small by comparison with the remaining terms containing transverse derivatives ($\delta \ll 1$). Taking account of these terms does not produce any qualitatively new results and leads only to a more laborious computation.

If in (4.7)-(4.9) we ignore the derivatives, we can obtain, as a first approximation, equations describing the homogeneous stationary state. Because the increments and the transverse inhomogeneities are small, these equations can be used to transform the terms in (4.7)-(4.9) containing derivatives. If we take this and the expressions for $\varkappa\nu$ from (4.9) and ν from (4.8) into account, we can write (4.7) as

$$\varepsilon G\frac{\partial I}{\partial\tau} - \varepsilon\frac{I}{g}\frac{\partial U}{\partial\tau} - g\nabla_\perp'[p(I)(\psi_1 + c)\nu\nabla_\perp' I] + \\ + \varepsilon\frac{\nu}{1-I}\nabla_\perp'[p(I)\nu\nabla_\perp' I] - I]F(U, I) - \varepsilon] = 0 \tag{4.10}$$

Here, in accordance with (3.1) and (3.3), and assuming that the terms containing transverse derivatives are small, we can put $\nu = fI/\varepsilon$. In (4.10) we neglect the squares of the slopes and assume that the perturbations do not change the total current, as a result of which $\partial U/\partial\tau = 0$; then we obtain

$$\varepsilon G\frac{\partial I}{\partial\tau} - p(I)A(I)I\Delta_\perp' I - I[F(U, I) - \varepsilon] = 0, \quad A(I) = \psi_1 + c - \frac{f}{g}\frac{I}{1-I} \tag{4.11}$$

Equations (4.10) and (4.11) describe a low-voltage arc discharge which is inhomogeneous in the transverse coordinates.

5. As was shown above, instability of the perturbations which are homogeneous with respect to y, z leads to an overthrow of the discharge into a stable part of the volt-ampere characteristic. Consider small perturbations in which the wave vector is directed along the plasma layer

$$I = I_0 + I'(t)\ \exp\ (i\mathbf{k}_\perp\mathbf{r}_\perp) \tag{5.1}$$

In the linear approximation, from (4.11) we have

$$\frac{dI'}{d\tau} = \frac{I_0}{\varepsilon G}[-r_d(g+f) - k_\perp^2 pA]I' \tag{5.2}$$

where the differential resistance is

$$r_d = \frac{1}{I}\left(\frac{1}{1-I}f - \frac{a}{I}g\right)\frac{1}{f+g} \tag{5.3}$$

We shall consider a region near the turning point of the volt-ampere characteristic ($r_d \approx 0$). In this case

$$I_0 > 0.5, \qquad \frac{f}{g}\frac{I_0}{1-I_0} \approx a \approx 1$$

and, since $\psi_1 + c \gg 1$, we find that $p > 0$, $A > 0$. Hence the term proportional to $k_\perp^2$ is positive and leads to damping. We note that this damping occurs not as a result of diffusion or thermal conductivity, but due to the appearance of transverse currents causing a redistribution of the Joule heat. The first term in brackets in (5.2) for $r_d < 0$ gives an increment the nature of which was studied above. As we see from (5.2), in the lower part of the volt-ampere characteristic ($r_d < 0$) the perturbations are unstable when $k_\perp$ is sufficiently small. The wavelength of the unstable perturbations satisfies the inequality

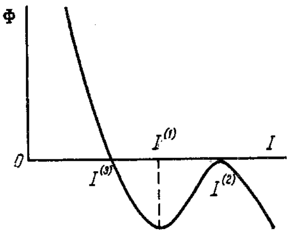

Fig. 3

$$\lambda > \lambda_* = 2\pi \sqrt{d} \left(\frac{A(I)\, p(I)}{|r_d|(f+g)} \right)^{1/2} \tag{5.4}$$

Since r_d is small and $\sqrt{d} > l$, in the whole region of unstable wavelengths, including the boundary ($\lambda = \lambda_*$), the assumption made in deriving (4.10), (4.11), that the transverse slopes were small, is justified.

Thus, in the lower part of the volt–ampere characteristic, a discharge which is homogeneous with respect to y, z is unstable for perturbations with $\lambda > \lambda_*$ irrespective of the resistance of the external circuit. As a result, in the range in which the resistance is negative, for sufficiently large electrodes there must develop a discharge which is inhomogeneous in the transverse direction and which is described by the nonlinear equation (4.11). Below we consider stationary solutions of this equation. When $\partial/\partial t = 0$, it follows from (4.11) that

$$\Delta'_{\perp} I = -\frac{d\Phi(I)}{dI}, \qquad \Phi(I) = \int \frac{F(U, I) - \varepsilon}{A(I)\, p(I)}\, dI \tag{5.5}$$

The extrema of the function $\Phi(I)$ correspond to points on the volt–ampere characteristic (3.2), there being a maximum on the upper branch [$I^{(2)}$] and a minimum on the lower $I^{(1)}$ (Fig. 3).

In the one-dimensional case, (5.5) has the form of the equation of motion of a point in the potential field $\Phi(I)$. It has a singularity of type "center" for $r_d < 0$ and of type "saddle" for $r_d > 0$. Thus, after passing through the turning point of the volt–ampere characteristic, when the homogeneous state becomes unstable, it is possible that a new stationary state of one of the following two forms is established:

1) Against the homogeneous background of the discharge with current density $I = I^{(2)}$ there is a wide region with reduced current density "soliton quenching" corresponding to the separatrix in the phase plane), the current at the minimum [$I^{(3)}$] being less than the current [$I^{(1)}$] on the lower branch of the volt–ampere characteristic (for given U);

2) the current density oscillates along the electrodes about the value $I^{(1)}$, so that the discharge has a transverse periodic structure.

For small perturbations the soliton solution has the form

$$I = I^{(2)} - (I^{(2)} - I^{(3)})\, \mathrm{sch}^2 (y / \Delta) \tag{5.6}$$

where

$$I^{(2)} - I^{(3)} = \frac{3}{2} I^3 (1 - I) \frac{f+g}{a} r_d \Big|_{I=I^{(2)}} \tag{5.7}$$

$$\Delta = \frac{8}{\pi} l \left[\frac{p(I)\, A(I)}{f+g} \ln\left(\frac{4}{\pi\delta}\right) \frac{1}{r_d} \right]^{1/2} \Bigg|_{I=I^{(2)}} \gg l \tag{5.8}$$

Although we have only considered the one-dimensional case above, we may hope that the cylindrically symmetrical solution also has a similar character. The stability of these new stationary states still has to be investigated.

It is possible that the formation of laces which are observed experimentally when the discharge is quenched [7] and the periodic transverse discharge structure announced in [6] are the results of the development of the above-mentioned instabilities.

LITERATURE CITED

1. A. V. Gurevich, "Electron temperature in a plasma in a variable electric field," Zh. Éksp. i Teor. Fiz., 35, No.2 (1958).
2. B. B. Kadomtsev, "The hydromagnetic instability of a plasma," in: Problems in Plasma Theory, Vol. 2 [in Russian], Atomizdat, Moscow (1963).
3. A. F. Volkov and Sh. M. Kogan, "The emergence of an inhomogeneous current distribution in semiconductors with negative differential conductivity," Zh. Éksper. i Teor. Fiz., 52, No. 6 (1967).
4. I. P. Stakhanov and I. I. Kasikov, "The volt–ampere characteristics of a low-voltage arc discharge," Zh. Tekhn. Fiz., 39, No. 8 (1969).

5. V. A. Zherebtsov, and I. P. Stakhanov, "The stability of a low voltage arc discharge," Zh. Tekhn. Fiz., 40, No. 12 (1970).
6. V. I. Debrilov, D. V. Karetnikov, N. P. Kosyreva, A. F. Nastoyashchii, and V. B. Turundaevskii, "The development of a low-voltage arc in a thermo-emissive diode with extensive electrodes," in: The Thermo-Emissive Transformation of Energy [in Russian], VNIIT, Moscow (1969).
7. G. A. Dyuzhev, A. M. Martsinovskii, G. E. Pikus, É. B. Sonin, and V. G. Yur'ev, "Some features of the volt–ampere characteristics of thermo-emissive transformations in an arc regime," Zh. Tekhn. Fiz., 37, No. 10 (1967).